

- (b) Define a cyclic group. Show that the  $n^{\text{th}}$  order roots of unity i.e.  $\exp(i2\pi k/n)$  for  $1 \leq k \leq n$  form a cyclic group of order  $n$  under scalar multiplication.

10

8. (a) Explain what is Isomorphism and Homomorphism.

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- (b) Explain how you obtain the direct product of two groups of orders  $h$  and  $k$ .

5

- (c) Define classes and subgroups of a group.

5

Exam. Code : 209001

Subject Code : 4869

M.Sc. Physics 1<sup>st</sup> Semester (Batch 2021-23)

PHY-402 : MATHEMATICAL PHYSICS

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt *five* questions in all, selecting at least *one* question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Plot graph of the following function,  $f(x)$  in the interval  $[-2\pi, 2\pi]$  :

$$f(x) = K \quad -\pi/2 < x < 0$$

$$= -K \quad 0 < x < \pi/2$$

$$\text{period of function} = 2\pi. \quad 5$$

- (b) Develop the Fourier series expansion of  $f(x) = (x+\pi)$  in the interval  $(-\pi, \pi)$ . Given period of function =  $2\pi$ . 5

- (c) Find the Fourier transform of

$$f(x) = 1 - x^2 \quad |x| < 1$$

$$= 0 \quad |x| > 1$$

10

2. (a) Define symmetric, anti-symmetric and mixed tensors. 5
- (b) Find the expression of a divergence of a vector function,  $A$  in spherical polar co-ordinates. 10
- (c) Given force,  $F$  acting on a two-dimensional linear oscillator may be described by  $F = -ikx - jky$ , where  $i$  and  $j$  are the unit vectors along  $X$  and  $Y$  axes respectively and  $k$  is the spring constant. Calculate the work done by this force when going from  $(1, 1)$  to  $(4, 4)$  along the line  $x = y$ . 5

#### SECTION—B

3. (a) Using Frobenius method solve the differential equation :

$$x(1-x) \frac{d^2y}{dx^2} + 2(1-2x) \frac{dy}{dx} - 2y = 0. \quad 10$$

- (b) Obtain the integral representation of Bessel's functions,  $J_1(x)$ . 10
4. (a) Solve Legendre's differential equation given below for positive integer,  $n$  and obtain general expression for Legendre's polynomials,  $P_n(x)$  :

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0. \quad 10+5=15$$

- (b) Using the definition of gamma function,  $\Gamma(n)$ , evaluate the integral :

$$\int_0^{\infty} x^{n-1} e^{-k^2 x^2} dx. \quad 5$$

#### SECTION—C

5. (a) Find the Laurent series expansion of  $F(z) = \frac{1}{z^2 + 2 - 3z}$  in the region (i)  $1 < |z| < 2$  (ii)  $0 < |z-1| < 1$ . 10

- (b) Find the poles for  $F(z) = \frac{z^4}{(z-4)(z-1)^2(z-3)}$ . Also, calculate the residue of  $F(z)$  at  $z = 1$ . 3+2=5

- (c) By use of the residue theorem, evaluate :

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}. \quad 5$$

6. (a) Evaluate  $\oint (z-3)^{-1} dz$  where the close path of integration is  $|z-3| = 2$  in the counter clockwise direction. 5
- (b) State and prove Cauchy's integral formula. 10
- (c) Integrate  $\frac{z^2+1}{z^2-1}$  along a circle of radius 1 with centre at  $z = i$ . 5

#### SECTION—D

7. (a) Show that the following sets are groups under the given laws of composition :
- (i) the set of all non-zero rational number under multiplication
- (ii) the set of all even integers under addition. 10